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THE PRICE CEILING THAT MINIMIZES AN EXPORTER'S PROFITS WITHOUT RAISING THE BUYERS' PRICE EXCESSIVELY

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ABSTRACT

Migration Shocks and Voting: Evidence from Ukrainian Migration to Poland

To reduce Russia's ability to fund its war in Ukraine, Western governments imposed a price ceiling on Russian seaborne oil exports. Policy-makers sought a ceiling level to lower Russia's oil profits without raising excessively the world price buyers pay for oil. Previous analyses have explored this problem using simulations and, with a single exception, have treated the non-Russian supply response as exogenous. We pose the problem theoretically as a constrained minimization problem of the policy maker and solve it, treating Russia as either a monopolist or an oligopolist facing heterogeneous rivals with endogenous supply.

JEL CLASSIFICATION: L13, Q41, D78
KEYWORDS: Dominance, Sanctions, Price cap, Ceiling evasion, Shadow fleet, Cournot oligopoly, Heterogeneous costs

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1 Introduction

There is a growing literature on determining the price ceiling on Russian oil exports to reduce funding of the Ukraine war. These consist largely of simulation models. We contribute to this literature by posing and solving theoretically the central policy problem: how to minimize Russia's oil profits subject to the constraint that the world price not be raised beyond some pre-specified bound. We begin by considering this constrained minimization problem in the simplest possible context, monopoly. In doing so, we extend the classic monopoly analysis of Arthur C. Pigou (1920) and Joan V. Robinson (1933), formulated almost a century before the Ukraine invasion. Next, we consider our constrained minimization problem in the context of the Turner-Sappington duopoly model (2024). Their model adds realism by taking account of (1) non-Russian oil production albeit from a single producer and (2) Russian evasion of the cap using the shadow fleet. Finally, we consider our profit-minimization problem in a setting that allows Russia to evade the ceiling by using a shadow fleet while other producers with heterogeneous costs observe the ceiling imposed on Russia and respond strategically.

Our paper is the first to analyze this constrained minimization problem in the presence of strategic interactions. We are able to address this problem in a systematic way only because our reaction-function approach to the Turner-Sappington duopoly model simplifies analysis and facilitates generalization. Formally, the policy-maker chooses a price ceiling to minimize $\Pi^R(\bar{p})$ subject to the constraint $P(\bar{p}) \leq W$, where the two functions, Russian profit Π^R and the buyers' price P, depend on the price ceiling $\bar{p}$. The policy-maker refrains from setting any ceiling that raises the world price above $W > 0$, which is taken as exogenous.

These two functions of the price ceiling are implicit in any model predicting the equilibrium effects on Russian profits of a price ceiling on Russian oil. In the market structures we examine, each function is badly behaved. The price function $P(\bar{p})$ in each market structure is kinked and non-monotonic. As for the profit function $\Pi^R(\bar{p})$, it is not only kinked in each market structure but, apart from the monopoly case, non-monotonic. Despite these apparent technical hurdles, we show that our constrained minimization problem can be easily solved, and the policy-maker's optimal ceiling can be characterized both analytically and graphically.

We prove that under each market structure the optimal ceiling can never occur at points in the price-ceiling domain where these technical difficulties arise. Over the rest of the domain, (1) the price function is monotonically decreasing; and (2) the profit function is monotonically increasing. At the optimum, therefore, further tightening of the price ceiling would lower Russian profits further but would raise the world price to an unacceptable level.

The following dominance property, which holds in each market structure we examine, simplifies our analysis: There exists a cutoff ceiling such that any price ceiling above this cutoff has a unique counterpart ceiling below the cutoff that results in the same equilibrium market price but nonetheless a strictly lower Russian profit. This crucial property is easy to prove and understand in the monopoly case but can be shown to hold as well when Russia has rivals and is able to evade the ceiling.

While our analysis exploits this common element in the Pigou-Robinson monopoly model and the oligopoly model, it is important to recognize a distinguishing feature of the oligopoly models. The effect of the price ceiling on Russian profits and the world price depends in part on the supply response of the non-Russian oil producers. However, in every previous study *except* Turner-Sappington's, this supply response has been treated as exogenous. It is by definition zero in Robinson's static monopoly model and by assumption zero in Johnson, Rachel, and Wolfram's (2023) pioneering static analysis of the price cap. In Kilian, Rapson, and Schipper's (2025) static model of the 2022 oil embargo, non-Russian supply is given by an exogenous function. The same assumption is made in Gars, Spiro, and Wachtmeister's (2025) static distributional analysis of Russia's oil-export control. In Cardoso, Salant, and Daubanes' (2026) dynamic sanction assessment model where Russia optimally expands its shadow fleet, they assume non-Russian supply to be either price-insensitive or given by an exogenous function. Finally, in Johnson, Rachel, and Wolfram's (2025) dynamic stochastic analysis of Russia's response to the price cap, they assume a price-insensitive random supply. While sensitivity of non-Russian supply can always be hypothesized, only in the Turner-Sappington duopoly model and our generalization to heterogeneous oligopoly is the response of non-Russian exports to a tighter ceiling endogenous. This seems important since non-Russian oil producers collectively produce around 90% of the world supply. As we show, it is this endogenous supply of the non-Russian producers that makes Russian profits as a function of the ceiling non-monotonic.

We contribute to the foundation laid by Turner and Sappington (2024) in two ways. First, we show that the non-monotonicity of Russian profits which they emphasize need not concern a policy-maker seeking to minimize Russian profits subject to a price constraint. More importantly, we show how to determine the optimal price ceiling when Russia can evade the sanction using its shadow fleet and faces other oligopolists with heterogeneous costs as well as a competitive fringe. This provides a realistic foundation for future empirical research on the quantitative design of a price-cap sanction.

We proceed as follows. In Section 2, we address our constrained profit minimization problem treating Russia as a monopoly. In Section 3, we address the same problem in Turner-Sappington duopoly model. Section 4 demonstrates that our analysis carries over to an oligopoly. Section 5 concludes.

2 The Optimal Ceiling in the Pigou-Robinson Monopoly Model

Joan Robinson (1933) considered a monopolist with increasing marginal costs subject to a price ceiling. She assumed the monopolist's cost function, denoted $C(q)$, has the following properties: $C(0) = 0$, $C'(0) > 0$, $C''(q) > 0$. To simplify, we assume that the marginal cost function and inverse demand function are linear, with respective vertical intercepts p_0 and p_{choke}. Since Robinson was generalizing the results that Arthur Pigou (1920) had obtained for this linear version, we refer to their results henceforth as the Pigou-Robinson analysis of monopoly constrained by a price ceiling.

If the price ceiling is above the monopoly price, the ceiling does not bind and the monopolist charges buyers the monopoly price. For any price ceiling below the monopoly price but above the point where the inverse demand curve and marginal-cost curve intersect (with vertical coordinate denoted p_d), the monopolist produces what is demanded at the ceiling price ($D(\bar{p})$). Hence, since the demand curve is downward-sloping, the monopolist produces a *larger* output when the ceiling is reduced. This counterintuitive output expansion occurs because, when the binding ceiling is lowered, the marginal revenue jumps up from below to above marginal cost for all outputs between the old optimum and the new one. For any price ceiling below p_d, however, the monopolist produces where the price ceiling intersects the marginal-cost curve. Since the marginal-cost curve is upward-sloping, the monopolist produces a *smaller* output when the ceiling is reduced. Hence, the monopolist's output is largest when the price ceiling equals p_d.

When the binding price ceiling exceeds p_d, there is no excess demand and the price buyers pay is received in its entirety by the seller: $P = \bar{p}$. On the other hand, when the price ceiling is below p_d, demand at the ceiling price exceeds what the monopolist supplies. Rationing results in a buyers' price just high enough that demand is limited to the quantity the monopolist supplies: $D(P) = C'^{-1}(\bar{p})$. In this case, the monopolist does not receive $P - \bar{p} > 0$. It may be pocketed by middlemen, distributed to those awarded marketable ration tickets, or dissipated as waiting times.

Figure 1 illustrates the results noted by Pigou-Robinson as well as a dominance property that greatly simplifies analysis of our optimization problem. For any price ceiling above p_d there is a unique price ceiling below p_d that results in the same buyers' price (P) but nonetheless a *strictly lower* Russian profit. Because of this dominance result, the policy maker can disregard ceilings above p_d when searching for the optimal price ceiling. Exactly the same dominance result arises when Russia has one rival or multiple rivals and it permits the same simplification of the optimization problem of the policy-maker.

Figure 2 plots the buyers' price as a function of the price ceiling in the Pigou-Robinson model. It depicts the previous results from a different perspective. For any price ceiling smaller than p_0, the monopolist produces nothing and the buyers' price must choke off demand: $P = p_{\text{choke}}$. For any price ceiling larger than p_{mon}, the monopolist charges the monopoly price: $P = p_{\text{mon}} < p_{\text{choke}}$. If the price ceiling $\bar{p} \in (p_d, p_{mon})$, then the buyers' price coincides with the ceiling and hence increases linearly along the 45-degree line. If the price ceiling $\bar{p} \in (p_0, p_d)$, then the buyers' price exceeds the ceiling and, as the ceiling increases and the monopolist produces more, the buyers' price decreases linearly.[1]

For every ceiling $\bar{p}_1 \in (p_d, p_{\text{mon}})$ there is a unique ceiling $\bar{p}_2 \in (p_0, p_{\text{mon}})$ with the same buyers' price (P) but smaller Russian profits. Profits are defined as gross revenue minus production cost. Gross revenue is strictly smaller since the monopolist cannot capture rents from rationing. Production cost is the same since the monopolist produces the same quantity when the ceiling is $\bar{p}_1$ or $\bar{p}_2$.

Finally, the monopolist's profit, $\Pi^R(\bar{p})$ is strictly increasing for $\bar{p} \in (p_0, p_d)$ since the monopolist sets his marginal cost equal to the ceiling and the lower the price ceiling the smaller the area of the

[1]The downward-sloping segment is linear since $\bar{p} = C'(Q)$ and $P = P(Q)$, implying $\frac{dP}{d\bar{p}} = P'/C''$, a negative constant given the assumed linearity of inverse demand and marginal cost.

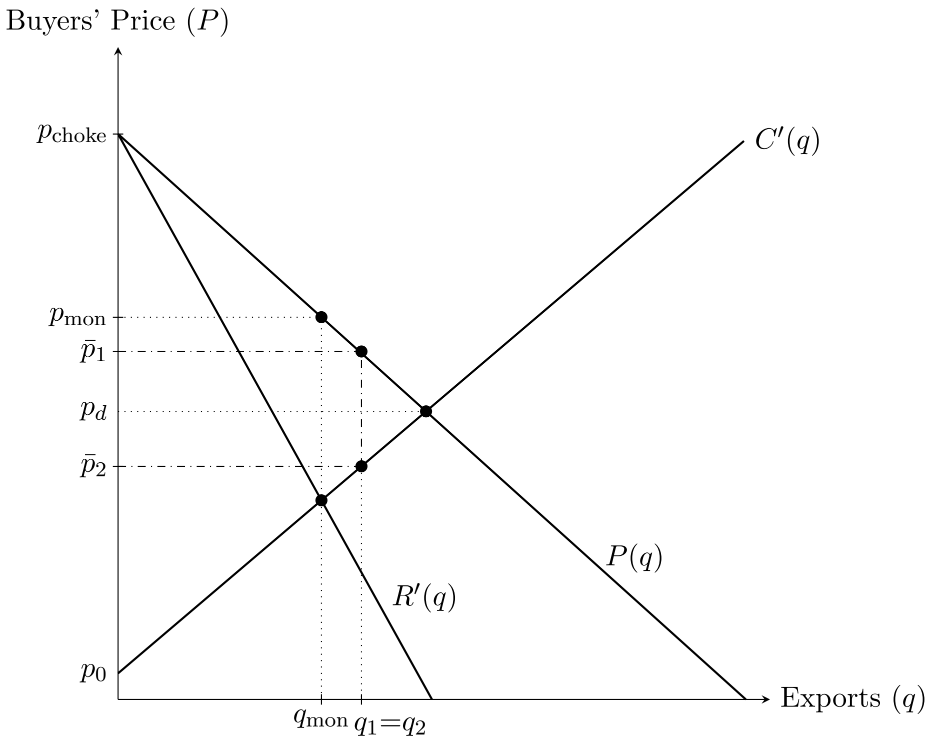

Figure 1: The diagram depicts (1) Pigou-Robinson's output expansion result that for any price ceiling $\bar{p} \in (p_d, p_{\text{mon}})$, tightening the ceiling raises monopoly production and (2) our dominance result that for any price ceiling in that interval (for example, $\bar{p}_1$) there is a unique ceiling below p_d ($\bar{p}_2$) where buyers pay the same price but Russian profits are strictly smaller.

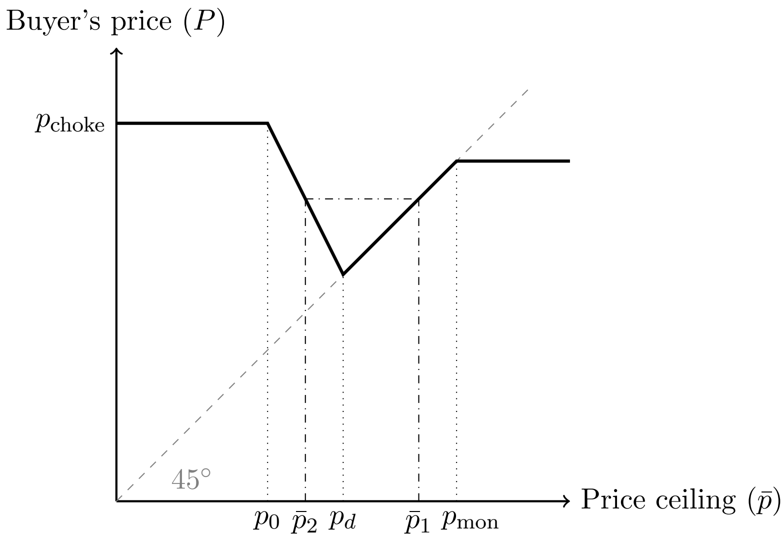

Figure 2: The price function, $P(\bar{p})$ in the monopoly model. The price function has two horizontal segments: a segment at height p_{choke} for $\bar{p} \in (0, p_0)$ and a lower segment at height p_{mon} for $\bar{p} \geq p_{\text{mon}}$. The dominance property insures that Russian profits are strictly smaller at $\bar{p}_2$ than at $\bar{p}_1$.

producer surplus "triangle."

Hence, to solve his optimization problem, the policy-maker simply finds the unique ceiling below p_d that results in a buyers' price of W. The optimal ceiling can be read directly from Figure 2. Analytically, the optimal price ceiling ($\bar{p}^*$) and associated exports uniquely solve $\bar{p} = C'(Q)$ and $P(Q) = W$, where W is exogenous and $W > \bar{p}^*$.

We now turn to Turner-Sappington's duopoly model and show the counterparts to these monopoly results.

3 The Optimal Ceiling in the Turner-Sappington Duopoly Model

In the Turner-Sappington model there are two firms with market power that choose outputs simultaneously. Russia chooses how much to sell (q_A) at the ceiling price and how much to sell (q_N) at the world price, which may be higher ($P \geq \bar{p}$). Because Russia is barred by the West from using the efficient Western services when it sells above the ceiling, higher production costs accompany the higher prices.

Turner-Sappington assume that the cost of non-Russian production is quadratic: $C(q) = cq +$

$.5kq^2$. That is, its marginal cost is linear with slope k and vertical intercept c, where $c > 0, k > 0$. They assume that demand is linear: $P = a - b(q + q_A + q_N)$, where $a > 0, b > 0$. The non-Russian exporter takes the aggregate Russian exports ($Q^R = q_A + q_N$) as given and maximizes his own profits:

$$\max_{q \geq 0} \Pi(q) \equiv P\left(Q^R + q\right) q - C\left(q\right).$$

Denote the reaction function of the non-Russian exporter as $B(Q^R)$.[2] If we plot $B(Q^R)$ with Q^R on the horizontal axis and q on the vertical axis, the reaction function decreases linearly with $|\text{slope}| = -B'(Q^R) = \frac{b}{2b+k} < 1/2$.

Notice that the reaction function of the non-Russian duopolist does not depend on the exogenous price ceiling. It follows that every Nash equilibrium of this simultaneous-move game results in a pair (Q^R, q) on this fixed non-Russian reaction function. The Nash equilibrium moves along $B(Q^R)$ because the *Russian* reaction function shifts in response to changes in the exogenous price ceiling ($\bar{p}$).

Turner-Sappington assume that the Russian cost function is $C^R\left(q_A, q_N\right) = c_A q_A + .5k_A q_A^2 + c_N q_N + .5k_N q_N^2 + .5k^R(q_A + q_N)^2$, where $c_N > c_A > 0$ and $k_N > k_A > 0$ and $k^R > 0$. Russia chooses q_A and q_N to maximize its profits given q and $\bar{p}$, thus solving

$$\max_{q_A \geq 0, q_N \geq 0} \Pi^R\left(q_A, q_N\right) \equiv p_m q_A + P\left(q_A + q_N + q\right) q_N - C^R\left(q_A, q_N\right), \text{ where } p_m = \min\left(\bar{p}, P(q_A + q_N + q)\right).$$

We focus on the case where it is optimal for Russia to sell some of its oil through each channel: $q_A > 0$, $q_N > 0$; for extremely tight ceilings, however $q_A = 0, q_N > 0$ is optimal.

In the Appendix, we show formally how the Nash equilibrium output pair changes as the ceiling tightens. We depict our conclusions in Figure 3, which may be explained as follows. When the ceiling just binds, the Nash equilibrium is at point I, the textbook Cournot point in the absence of a ceiling. We denote this ceiling $\bar{p}_{duop}^{A,N}$ since in the induced duopoly equilibrium Russia sells through both channels. As the ceiling tightens, the Nash equilibrium output pair moves from point I down the reaction function toward point II. This increase in Q^R occurs for the following reason. If there were no foreign response to a tightening of the ceiling, Russia would expand its aggregate exports

[2] Since Russian exports enter the rival's maximization problem as a sum, the rival's best reply is simply $B(q_A + q_N) = B(Q^R) = \dfrac{a - bQ^R}{2b + k}$.

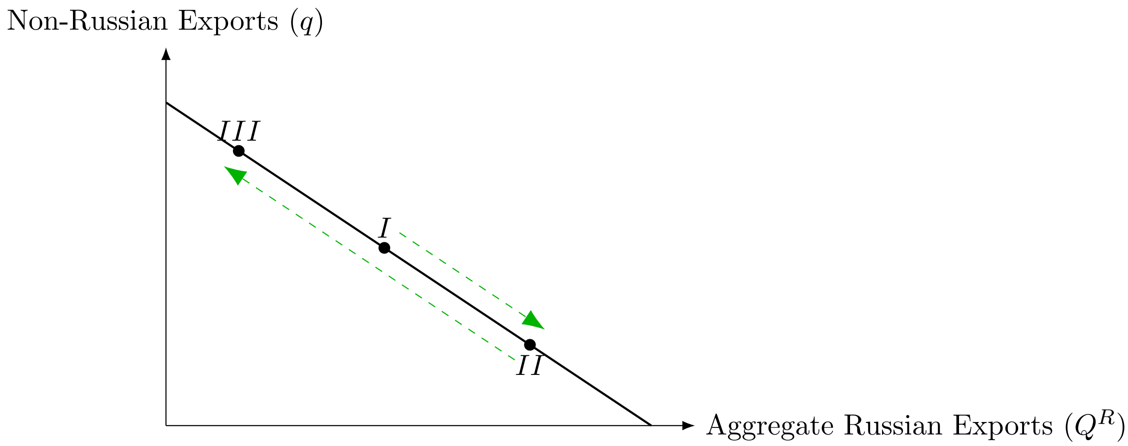

Figure 3: Loci of Nash equilibria along the non-Russian best-reply curve as the price ceiling is lowered. Starting from the Cournot equilibrium (I) when the ceiling just binds, the equilibrium moves first to II, then to III.

just like the monopolist in Pigou-Robinson's single-channel case. Because of strategic substitutes, however, this expansion by Russia causes a contraction in the rival's exports and that causes a further expansion in Russian exports and a further contraction in the rival's exports... Thus, the Pigou-Robinson effect is magnified.[3]

When the Nash equilibrium output pair reaches point II, Russian exports reach a maximum. We denote the ceiling inducing the equilibrium at II as $\bar{p}_d$, the counterpart to p_d in the monopoly model.[4] As shown in the Appendix, if the price ceiling is lowered below $\bar{p}_d$, the Nash equilibrium output pair retraces its steps and moves back up the reaction function. Intuitively if there were no foreign response, Russia would contract its aggregate exports when the ceiling is lowered just like the monopolist in Pigou-Robinson's single-channel case.[5] This Russian contraction would cause non-Russian producers to expand. When the Nash equilibrium output pair reaches point III, Russia sells only using the shadow fleet ($q_A = 0, q_N > 0$). Since it sells nothing at the ceiling, further reductions in the ceiling do not displace the equilibrium, and the output pair remains at point III. We denote the ceiling inducing this equilibrium as p^N_{duop}.

[3]If $q = B(Q^R)$ and $Q^R = R(\bar{p}, q)$ then, after differentiating the system we obtain -$\frac{dQ^R}{d\bar{p}} = -\frac{R_{\bar{p}}}{1-B'R_q} > -R_{\bar{p}} > 0$. If the ceiling induces an equilibrium between point I and point III, tightening the ceiling causes Russia's reaction function to shift to the right ($-R_{\bar{p}} > 0$).

[4]Turner and Sappington (2024) use this same notation $\bar{p}_d$ to denote the ceiling inducing the maximum Q^R.

[5]If the ceiling is below $\bar{p}_d$ Russia's reaction function shifts to the left ($-R_q < 0$) as is shown in the Appendix.

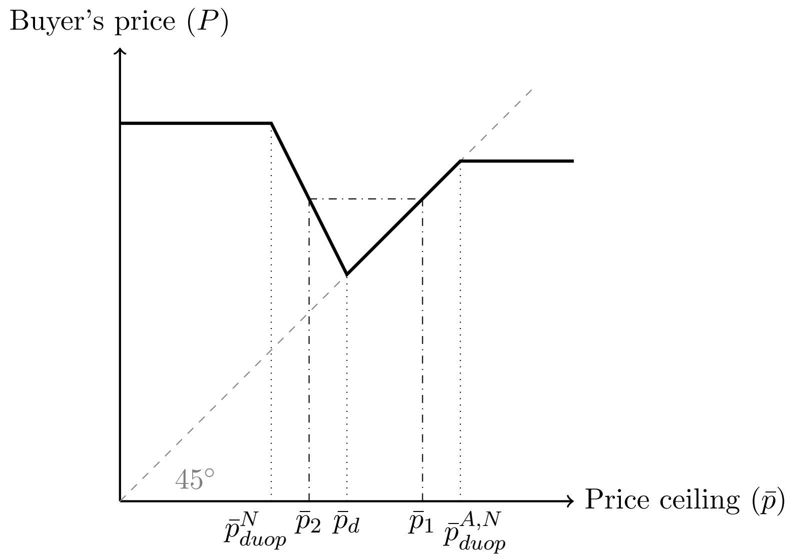

Figure 4: Note that the shape of this function is identical to the function in Figure 2.

Since the downward-sloping reaction function is flatter than the -45-degree line (|slope| $= -B'(Q^R) < 1/2 < 1$) output pairs further down the non-Russian reaction function have a larger total supply ($Q^R + q$) and therefore a lower world price (P). In Figure 4, we plot the price buyers pay as a function of the ceiling price. Note that the shape is identical to the price function in the Pigou-Robinson model (see Figure 2), when the latter is specialized to linear demand and marginal cost. As in the Pigou-Robinson case, the buyer's price coincides with the 45-degree line for ceilings larger than $\bar{p}_d$ and exceeds that line for ceilings smaller than $\bar{p}_d$.

We now show that the price ceiling that achieves the constrained minimum in Russian profits can never exceed the threshold $\bar{p}_d$. As Figure 4 depicts, every ceiling larger than $\bar{p}_d$ has a unique counterpart with the same output pair (Q^R, q) and world price (P). In the equilibrium induced by the larger ceiling, Russia earns the same price per barrel in each channel ($P = \bar{p}_1$) and incurs the same infra-marginal losses ($q_N P'$). So in the equilibrium associated with ceiling $\bar{p}_1$, Russia maximizes its aggregate profits by selling in the two channels in a way which minimizes its production costs. The most profitable way to sell Q^R is to equate the marginal cost of production in the two channels.

In the equilibrium induced by the smaller ceiling ($\bar{p}_2$), Russia sells the same aggregate output as at $\bar{p}_1$. While it sells part of its output at the same price P, it sells the remainder of its output at the ceiling price, which is strictly smaller ($\bar{p}_2 < P = \bar{p}_1$). Hence, Russia earns a smaller gross revenue when the ceiling is $\bar{p}_2$ rather than $\bar{p}_1$ just like in the monopoply case.

The cost of producing the common output is weakly larger when the ceiling is $\bar{p}_2$. This follows since when the ceiling is $\bar{p}_1$, the cost of producing the common output is minimized. It follows that Russia earns a strictly smaller profit when the ceiling is $\bar{p}_2$ rather than $\bar{p}_1$.[6] Consequently, in solving the constrained profit minimization problem, the policy-maker need not consider ceilings larger than $\bar{p}_d$.

Having concluded that the policy-maker can restrict attention to ceilings where $\bar{p} \leq \bar{p}_d$, we focus on Russia's maximization problem when facing a price ceiling $\bar{p} \leq \bar{p}_d$. We show that for any such price ceiling, Russia's equilibrium profits are strictly increasing in the price ceiling: $\dfrac{d\Pi^R}{d\bar{p}} > 0$.

Russia chooses q_A and q_N to maximize its profits given q and $\bar{p} \leq \bar{p}_d$, thus solving

$$\Pi^R = \max_{q_A \geq 0, q_N \geq 0} \bar{p} q_A + P\left(q_A + q_N + q\right) q_N - C^R\left(q_A, q_N\right).$$

Hence,

$$\begin{aligned} \bar{p} + q_N P' - C^R_A(q_A, q_N) &= 0 && (1) \\ P(q_A + q_N + q) + q_N P' - C^R_N\left(q_A, q_N\right) &= 0, && (2) \end{aligned}$$

where the subscripts on the cost function denote respective partial derivatives: $C^R_A(q_A, q_N) \equiv \frac{\partial C^R}{\partial q_A}$ and $C^R_N(q_A, q_N) \equiv \frac{\partial C^R}{\partial q_N}$.

In the Nash equilibrium,

$$q = B(q_A + q_N). \qquad (3)$$

[6] To establish this dominance property formally, denote with the subscript 1 variables associated with the larger ceiling and denote with the subscript 2 variables associated with the smaller ceiling: $\bar{p}_1 > \bar{p}_2$. Since the two ceilings are "non-identical twins": $Q^R_1 = Q^R_2 = Q^R$, $q_1 = q_2 = q$, $P_1 = P_2 = P$, $\bar{p}_1 = P_1 = P$, $\bar{p}_2 < P_2$. Consider the difference between Π^R_2 and Π^R_1 by evaluating Π^R when $\bar{p} = \bar{p}_1$ and when $\bar{p} = \bar{p}_2$. That is,

$$\begin{aligned} \Pi^R_2 - \Pi^R_1 &= P_2 q_{2,N} + \bar{p}_2 q_{2,A} - C^R(q_{2,A}, q_{2,N}) - P_1 q_{1,N} - \bar{p}_1 q_{1,A} + C^R(q_{1,A}, q_{1,N}) \\ &= -P[Q^R - q_{1,A}] + P[Q^R - q_{2,A}] - P q_{1,A} + \bar{p}_2 q_{2,A} + C^R(q_{1,A}, q_{1,N}) - C^R(q_{2,A}, q_{2,N}) \\ &= -q_{2,A}[P - \bar{p}_2] - [C^R(q_{2,A}, q_{2,N}) - C^R(q_{1,A}, q_{1,N})] < 0. \end{aligned}$$

Each of the two sets of terms in square brackets in the last expression is positive, so $\Pi^R_2 < \Pi^R_1$. The first is strictly positive since $\bar{p}_2 < \bar{p}_d$ implies $P > \bar{p}_2$. As for the second set of terms in square brackets, it is the difference of two costs of producing the same aggregate exports (Q^R), and $C^R(q_{1,A}, q_{1,N})$ is the minimum of these costs.

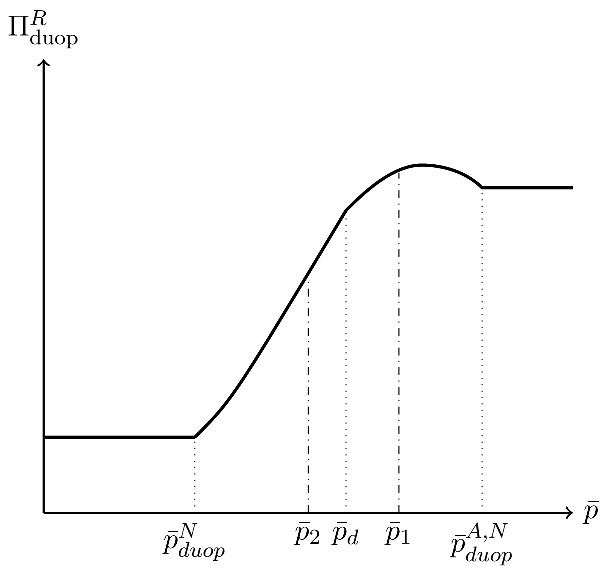

Figure 5: Russia's equilibrium profits in the duopoly model as a function of the price ceiling. The horizontal segment on the right is Russia's aggregate profits when the ceiling is not binding and using both channels is Russia's equilibrium response; the lower segment on the left is Russia's profits when the ceiling is so tight that Russia exports nothing at the ceiling price ($q_A = 0, q_N > 0$). As Turner-Sappington emphasized, there is a local maximum for some $\bar{p} > \bar{p}_d$. But this plays no role in our constrained optimization problem: The optimal ceiling must be smaller than $\bar{p}_d$ since for any larger ceiling like $\bar{p}_1$ there is a unique smaller ceiling ($\bar{p}_2$ in this case) where Russia earns a strictly smaller profit.

Substituting for q and totally differentiating, we obtain:

$$\frac{d\Pi^R}{d\bar{p}} = q_A+q_N P'B'(\frac{dq_A}{d\bar{p}}+\frac{dq_N}{d\bar{p}})+[\frac{dq_A}{d\bar{p}}(\bar{p}+q_N P'-C^R_A)+\frac{dq_N}{d\bar{p}}(P(B(q_A+q_N)+q_A+q_N)+q_N P'-C^R_N)] > 0. \tag{4}$$

The inequality follows since (i) the terms in square brackets are zero from equations (1)-(2); (ii) $q_A > 0$ and $P'B' > 0$; and, (iii) for $\bar{p} < \bar{p}_d$ when the price ceiling falls, the equilibrium point moves up the reaction function and Q^R also falls ($\frac{dq_A}{d\bar{p}} + \frac{dq_N}{d\bar{p}} > 0$).[7] The property that Russian profits become strictly lower as the cap $\bar{p} \leq \bar{p}_d$ becomes tighter is reflected in Figure 5.

Since reducing the ceiling strictly reduces Russian profits and strictly increases the world price

[7]Note that when the ceiling first binds, $\bar{p} > \bar{p}_d$ and equation (4) implies that $\frac{d\Pi^R}{d\bar{p}} = q_A + q_N P'B'(\frac{dq_A}{d\bar{p}} + \frac{dq_N}{d\bar{p}})$. But since the last factor of the second term is negative, the product of the four factors of the second term is negative. Since the second term will be negative, it may (and under Turner-Sappington's assumptions does) offset the first term resulting in Russian profits rising when the price ceiling first binds. This effect, which Turner-Sappington emphasize, can never occur under monopoly whether the monopolist has one channel or many since it requires a contraction in non-Russian exports when the ceiling first binds. Russia would see such a contraction as an outward shift in its residual demand and that may raise its profits. By appealing to the dominance property, we can disregard all such ceilings when solving the constrained profit-minimization problem of the policy maker.

when $\bar{p} \le \bar{p}_d$, the constrained minimum occurs where $P(\bar{p}) = W$ as in the monopoly model. To deduce the unique optimal price ceiling $\bar{p}^*$ and the three production variables at the optimum (q_A, q_N, and q), solve simultaneously the following four equations (1)-(3) and $P(q + q_A + q_N) = W$ and verify that $W > \bar{p}^*$.

4 The Optimal Ceiling in an Oligopoly Model with Heterogeneous Costs

Suppose that, in addition to Russia, there are F non-Russian strategic exporters with heterogeneous costs. Assume firm i has quadratic cost $C_i = c_i q_i + .5kq_i^2$ for $i = 1, \ldots, F$. Index the firms so that a firm with a lower index has a weakly lower marginal cost intercept (c_i): $c_1 \le c_2 \ldots \le c_F$. Suppose $\alpha \le F$ firms are active in equilibrium. If $P - c_i \le 0$, the firm must be inactive ($q_i = 0$). If $P - c_i > 0$, the firm must be active ($q_i > 0$). Since all firms sell at the world price (P), it follows that the active firms will be those with the lowest marginal-cost intercepts: firms $1, 2, \ldots, \alpha$. Assuming firm i is active, its exports $q_i > 0$ satisfy the following condition:

$$P(q + Q^R) + q_i P' - c_i - kq_i = 0 \tag{5}$$

where $\sum_{i=1}^{F} q_i = q$. Maintaining the assumption that demand is linear[8] and differentiating, we obtain:

$$P'dq + P'dQ^R + (P' - k)dq_i = 0. \tag{6}$$

Summing over the α active firms, we obtain: $-\dfrac{dq}{dQ^R} = \dfrac{-\alpha P'}{k - (\alpha + 1)P'}$. It follows that when there are α active firms, an increase of one unit in Q^R results in a constant reduction in the rival exports of less than 1. As a result, the equilibrium price buyers pay falls. If it falls below the marginal cost intercept of some active non-Russian firm, that firm becomes inactive and α is reduced by 1. The smaller the number of active firms the smaller the constant |slope|. That is, the "reaction function" $q(Q^R)$ is piecewise linear and convex with the |slope| of its steepest segment less than 1. This function does not shift when the ceiling changes.

[8] Although we focus the analysis on the oligopolists, the model can be readily extended to accommodate a fringe of competitive producers. Our model's demand for the oligopolists' oil may be interpreted as a residual demand, i.e., net of a competitive fringe's linear supply.

If the ceiling is not binding, the equilibrium occurs where this reaction function intersects the Russian reaction function discussed in the Appendix. We labeled this equilibrium I in the previous section. As the ceiling tightens, the Russian reaction function shifts and the equilibrium moves down this convex reaction function. Since its |slope| is smaller than 1, the buyers' price will fall and it is possible that the non-Russian firm with the highest marginal cost intercept will become inactive. In that case, α is again reduced by 1, and the constant |slope| of the reaction function becomes flatter. Eventually, the ceiling tightens so much that Q^R is maximized. We denoted this as point II in the previous section. Further tightening of the ceiling will cause the export pair to move back up the rivals' reaction function. We denoted the ceiling where Russian equilibrium exports are largest as $\bar{p}_d$ in the duopoly section (and p_d in the monopoly section). To avoid confusion, let's denote it as $\hat{p}_d$ in this oligopoly section.

For any ceiling $\bar{p}_1$ larger than $\hat{p}_d$, there will be a unique smaller ceiling $\bar{p}_2 < \hat{p}_d < \bar{p}_1$ with the same P, Q^R, q. Russian profits will be strictly smaller at $\bar{p}_2$ as in the previous section, and hence the policy-maker can restrict his search to ceilings smaller than $\hat{p}_d$.

If the cost functions are identical among non-Russian exporters, then all will be active at the same time and their "aggregate" reaction function will be linear with a |slope| < 1. In that case, to determine the policy-maker's optimal choice, one determines the following four variables $\bar{p}^*, q, q_A, q_N$ from the two Russian first-order conditions (1) and (2), the one non-Russian first-order condition and $W = P(q_A + q_N + q)$.

If the cost functions are heterogeneous, one proceeds as follows. Given W, one first determines which firms are active in the optimal solution. Since at the constrained optimum $W = P$ every firm with $c_i \geq W$ must be inactive and every firm with $c_i < W$ must be active. One then solves for the $3 + \alpha$ variables $(\bar{p}^*, q_A, q_N, \{q_i\}_{i=1}^{\alpha})$ using the $3 + \alpha$ equations (α first-order conditions from the active firms, two first-order conditions for Russia (1) and (2), and $W = P(q_A + q_N + \sum_{i=1}^{\alpha} q_i)$

In each case, there may be two solutions to the equations. For example, in Figure 4, a horizontal line of height W may cross $P(\bar{p})$ twice and each solution satisfies the equations above. The solution where the buyers' price equals the price ceiling should be ignored since in that case the ceiling exceeds $\hat{p}_d$. Given our dominance result, the unique optimal solution will have $W > \bar{p}^*$.

5 Conclusion

The ideal price cap for policy makers minimizes Russian oil profits while avoiding an excessive increase in the global oil price. In this paper, we formally address this central problem: setting the price ceiling so as to minimize Russian profits under the constraint that the price of oil does not exceed a given level.

We are the first to pose and solve this problem in a setting that allows Russia to evade the ceiling by using a shadow fleet while other producers with heterogeneous costs observe the ceiling imposed on Russia and respond strategically.

Our reaction-function approach allows us to establish our dominance result. Using it, we show that the optimal cap lies in an interval in which the counterintuitive effects emphasized in Turner and Sappington (2024) never occur. This approach allows us to simplify their analysis and to extend it to the case where Russia faces more than one rival and non-Russian exporters have different costs of production. The most general model presented in Section 4 can readily capture a situation in which the U.S., Russia, Saudi Arabia, and other major producers each maximize profits while facing a residual demand net of a competitive fringe supply. Hence, our approach establishes the foundation for future applied research on the price cap sanction taking into account strategic interactions.

Appendix

Turner and Sappington (2024) parametrize the inverse demand function as

$$P(Q) = a - bQ \tag{7}$$

and cost functions as

$$C(q) = cq + \frac{k}{2}q^2 \tag{8}$$

$$C^R(q_A, q_N) = c_A q_A + \frac{k_A}{2}q_A^2 + c_N q_N + \frac{k_N}{2}q_N^2 + \frac{k^R}{2}(q_A + q_N)^2. \tag{9}$$

Turner and Sappington (2024) consider the case where the parameters have the values specified in Table 1. We adopt these values to determine the direction of effects and to plot the best-reply diagram in Figure 6.

Parameter	Parameter Value	Parameter	Parameter Value
a	163.33	b	1.03703×10^{-6}
c	2.5	k	6×10^{-7}
c_A	2.5	k_A	5×10^{-7}
c_N	5.0	k_N	1×10^{-6}
		k^R	1×10^{-7}

Table 1: Parameters in Turner and Sappington (2024).

To characterize comprehensively Russia's best responses over the unrestricted domain of ceilings, we reconsider Russia's profit-maximization problem, reframing the price of sanctioned exports as an additional choice variable subject to a pair of constraints:

$$\max_{p_m \geq 0, q_A \geq 0, q_N \geq 0} p_m q_A + P\left(q_A + q_N + q\right) q_N - C^R\left(q_A, q_N\right), \tag{10}$$

subject to:

$$p_m \leq \bar{p} \tag{11}$$

$$p_m \leq P(q_A + q_N + q). \tag{12}$$

Appending the multipliers λ and γ to the two inequality constraints, we form the Lagrangean

($\mathcal{L}$):

$$\mathcal{L} = p_m q_A + P\left(q_A + q_N + q\right) q_N - C^R\left(q_A, q_N\right) + \lambda\left[P\left(q_A + q_N + q\right) - p_m\right] + \gamma\left[\bar{p} - p_m\right]. \qquad (13)$$

When the three decision variables are strictly positive, Russia's profit-maximizing choices must satisfy the following conditions:

$$\begin{aligned}
p_m + q_N P' - C_A^R(q_A, q_N) + \lambda P' &= 0 && (14)\\
P(q_A + q_N + q) + q_N P' - C_N^R(q_A, q_N) + \lambda P' &= 0 && (15)\\
q_A - \lambda - \gamma &= 0 && (16)\\
\lambda \geq 0,\ P(q_A + q_N + q) - p_m &\geq 0 \text{ with comp. slackness} && (17)\\
\gamma \geq 0,\ \bar{p} - p_m &\geq 0 \text{ with comp. slackness,} && (18)
\end{aligned}$$

where the subscripts on the cost function denote respective partial derivatives: $C_A^R(q_A, q_N) \equiv \frac{\partial C^R}{\partial q_A}$ and $C_N^R(q_A, q_N) \equiv \frac{\partial C^R}{\partial q_N}$.

Russia's aggregate reaction function is continuous and, given the assumptions in the main body of this article, consists of linear segments. Its upper segment, associated with large exports of the rival, has a slope steeper than -1 and coincides with the textbook aggregate reaction function; its middle segment has a slope equal to -1; but its lower segment is positively sloped.[9]

To verify these claims, fix $\bar{p}$ and assume that q varies. If $\lambda > 0, \gamma = 0$ then, from (17) and (18), $p_m = P(q_A + q_N + q) \leq \bar{p}$. From (16), $\lambda = q_A$. Substituting out of λ in (14) and (15), we obtain:

$$\begin{aligned}
P(q_A + q_N + q) + (q_A + q_N)P' - C_A^R(q_A, q_N) &= 0 && (19)\\
P(q_A + q_N + q) + (q_A + q_N)P' - C_N^R(q_A, q_N) &= 0. && (20)
\end{aligned}$$

For any conjectured exports of the rival, these two equations define Russia's exports through each

[9] There is in addition a fourth linear segment where the equilibrium may occur. It arises when the price ceiling is so low that Russia sells nothing at the ceiling ($q_A = 0$). Along this segment $P(q + q_N) + q_N P' = C_N^R(0, q_N)$. Therefore, the segment is downward-sloping and does not shift when the ceiling is tightened. Hence, in equilibria along this segment, q_N is invariant to the cap level and defined by $P(B(q_N) + q_N) + q_N P' = C_N^R(0, q_N)$, where B is the non-Russian exporter's reaction function to Russian total exports, presented in the main text. Denote this q_N^*. In this case, $\lambda = 0$ and $\bar{p} \leq -q_N^* P' + C_A^R(0, q_N^*)$. Turner and Sappington denote this threshold as $p_0 = 41.82$ and display the constant q_N^* that arises for ceilings below this threshold in their Figure 1.

channel and hence their sum. Each first-order condition coincides with the textbook best-replies of an exporter who faces no price ceiling (or, equivalently, a non-binding one) and can sell through two channels, each with its channel-specific cost function.

If instead $\lambda > 0, \gamma > 0$, then from (17) and (18), $p_m = P(q_A + q_N + q) = \bar{p}$. Since $P(Q^R + q) = \bar{p}$, the slope of this segment is -1 ($\frac{dq}{dQ^R} = -1$).

Using (14)–(16), we can write the three linear equations determining $q_A, q_N,$ and $\gamma:$

$$P(q_A + q_N + q) + (q_A + q_N - \gamma)P' - C_A^R(q_A, q_N) = 0 \tag{21}$$

$$P(q_A + q_N + q) + (q_A + q_N - \gamma)P' - C_N^R(q_A, q_N) = 0 \tag{22}$$

$$\bar{p} = P(q_A + q_N + q). \tag{23}$$

If the conjectured rival exports are smaller, γ is larger and eventually $\gamma = q_A$, implying $\lambda = 0$ and $\gamma > 0$.

When $\lambda = 0$ and $\gamma > 0$, (17) and (18) imply $p_m = \bar{p} \leq P$, and the conditions defining the exports through each channel (q_A, q_N) reduce to:

$$\bar{p} + q_N P' - C_A^R(q_A, q_N) = 0 \tag{24}$$

$$P(q_A + q_N + q) + q_N P' - C_N^R(q_A, q_N) = 0. \tag{25}$$

For a given binding price ceiling, we can determine how Russian exports in each channel (and hence their sum) change when q changes. Under these conditions, it turns out that, when signed using the above parameter values, $\frac{dq_A}{dq} > 0, \frac{dq_N}{dq} < 0,$ and $\frac{dQ^R}{dq} > 0$. Hence, the aggregate reaction function is positively sloped.[10]

How does this aggregate reaction function shift when the ceiling is lowered? The segment with slope of -1 must shift out parallel since aggregate exports must increase to maintain the equality

[10] Given the assumed functional forms, $P' = -b$, $C_{AA}^R = k_A + k^R; C_{NN}^R = k_N + k^R; C_{AN}^R = k^R$. Totally differentiating (24) and (25), the system can be expressed as: $\begin{pmatrix} -C_{AA}^R & P' - C_{AN}^R \\ P' - C_{AN}^R & 2P' - C_{NN} \end{pmatrix} \begin{pmatrix} dq_A \\ dq_N \end{pmatrix} = \begin{pmatrix} 0 \\ -P' dq \end{pmatrix}$. Solving this system and plugging in parameter values, we conclude that $\frac{dq_A}{dq} = \frac{P'(P' - C_{AN}^R)}{\Delta} > 0$, and $\frac{dq_N}{dq} = \frac{P' C_{AA}^R}{\Delta} < 0$ where $\Delta = -C_{AA}^R(2P' - C_{NN}^R) - (P' - C_{AN}^R)^2$. Therefore $\frac{dQ^R}{dq} = \frac{P'(P' - C_{AN}^R + C_{AA}^R)}{\Delta} > 0$.

between the world price and the lowered ceiling. As for the positively-sloped segment, it shifts leftward if the ceiling is lowered. Interestingly, lowering the ceiling causes q_N to expand but ceiling sales to contract by more so that their sum falls.[11]

When the intersection point lies on the segment of Russia's aggregate reaction function with a slope steeper than -1, the cap strictly exceeds the world price and is not binding. Tightening the cap, therefore, has no effect on Russia's aggregate exports.

When the intersection point lies on the segment of Russia's aggregate reaction function with a slope of -1, a marginal decrease in the ceiling will cause the intersection point to move down the rival's best reply. In that case, Russia's aggregate exports increase, the rival's exports decrease, and the world price falls.

When the intersection point lies on the positively-sloped segment of Russia's aggregate reaction function, a marginal decrease in the ceiling will cause the intersection point to reverse direction and move back up the rival's best reply. In that case, Russia's aggregate exports decrease, the rival's exports increase, and the world price rises, diverging from the reduced price ceiling.

We illustrate each of these three cases in Figure 6, which is constructed using the parameters in Table 1. In this Figure, the intersection point for a just binding cap of $\bar{p}^{A,N}_{duop} \approx 71.52$ (denoted I in the diagram) is on the edge of the steeper negatively-sloped segment; since the cap is just binding, this equilibrium is equivalent to one with no sanctions. In addition, the intersection point for a cap of $\bar{p}_d \approx 56.35$ (denoted II) lies on the edge of the segment with a slope of -1. Lastly, the intersection point for a cap of $\bar{p}^{N}_{duop} \approx 41.86$ lies on the edge of the positively-sloped segment; any cap at or below this level leads to an equilibrium where no sales under the cap occur. Notice that point II is southeast of I, but III is northwest of both points.

[11] Totally differentiating with respect to $\bar{p}$ (holding q constant), we can write the system in matrix form as: $\begin{pmatrix} -C^R_{AA} & P'-C^R_{AN} \\ P'-C^R_{AN} & 2P'-C^R_{NN} \end{pmatrix} \begin{pmatrix} dq_A \\ dq_N \end{pmatrix} = \begin{pmatrix} -1d\bar{p} \\ 0 \end{pmatrix}$. Since $\Delta > 0$, $\frac{dq_A}{d\bar{p}} = \frac{C^R_{NN}-2P'}{\Delta} > 0$; $\frac{dq_N}{d\bar{p}} = \frac{P'-C^R_{AN}}{\Delta} < 0$; $\frac{dQ^R}{d\bar{p}} = \frac{C^R_{NN}-C^R_{AN}-P'}{\Delta} = \frac{k_N-P'}{\Delta} > 0$.

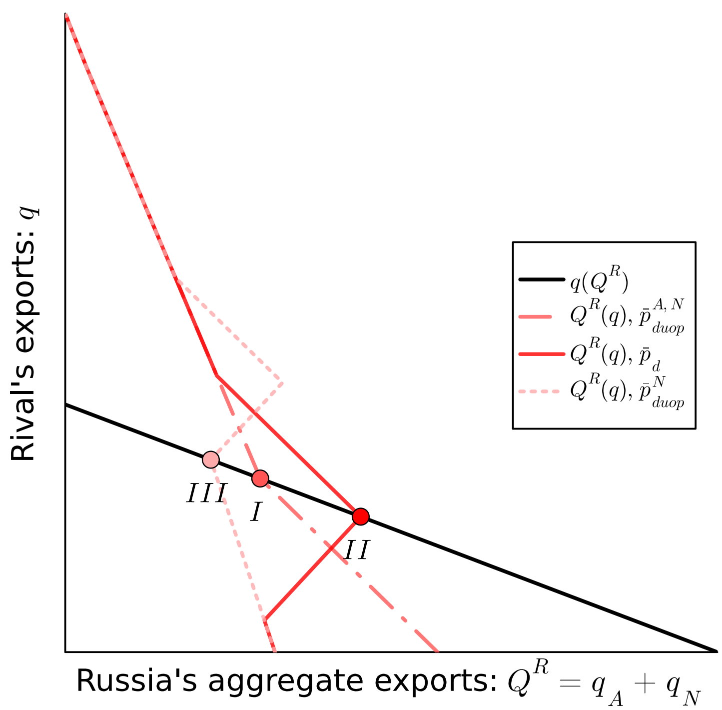

Figure 6: Three different cap levels result in different points of intersection between the rival's fixed reaction function and Russia's shifting reaction function. Intersection point I represents the case where the cap just binds at level $\bar{p}^{A,N}_{duop}$. Tightening the cap leads to an expansion in Russian exports up until a cap level $\bar{p}_d$. Further reductions in the cap contract Russian exports until a cap level $\bar{p}^{N}_{duop}$, beyond which no oil is sold at the ceiling price.

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